CHAPTER 8
SCREWS, FASTENERS, NONPERMANENT JOINTS

This chapter deals with the design and analysis of nonpermanent fasteners such as bolts, power screws, cap screws, setscrews, keys and pins.

8-1 Thread Standards and Definitions
A screw thread used for fastening and related terminology are shown below in the figure:

Another important dimension not shown in the figure is the lead \( l \), which is the distance that the nut advances along the screw axis when it is given one complete turn. We have \( l = np \), where \( n \) is the number of threads for multiple-threaded screws. Standard screws are single-threaded, and hence for them \( l = p \). There are two standards for screw threads, which are Unified (UN, Table 8-2, pg. 405) for inch-types and Metric (M, Table 8-1, pg. 404) for millimeter-types. In each type, there are Course (C) and Fine (F) threads. Furthermore, the MJ threads for the Metric and UNR threads for the Unified have rounded roots to improve their fatigue strengths. As an example, a M12 \( \times 1.75 \) thread would mean a Metric thread with a major diameter of \( d = 12 \) mm and a pitch of \( p = 1.75 \) mm. Also, a \( \frac{5}{8} \) in – 18 UNRF would indicate a Unified Fine thread with a rounded root having a major diameter of \( d = \frac{5}{8} \) in = 0.625 in and 18 threads in one inch of axial length.

8-2 Power Screws
The power screws are used for power transmission, and although the basic terms are same as those of the fastening screws, nevertheless they have different threads. There are two types of threads: Square and Acme, which are shown below. Preferred pitches for Acme threads are given in Table 8-3, pg. 406 of the textbook.
Important considerations for the power screws are the torques needed for raising a load \((T_R)\) and lowering a load \((T_L)\), which are calculated for a square thread as

\[
T_R = F \frac{d_m}{2} \left( \frac{\pi f d_m + l}{\pi d_m - fl} \right) \quad \text{and} \quad T_L = F \frac{d_m}{2} \left( \frac{\pi f d_m - l}{\pi d_m + fl} \right)
\]

where \(d_m\) is the mean diameter of the power screw given for a square thread as \(d_m = d - p/2\). Also, \(F\) is the load to be raised or lowered and \(f\) is the coefficient of friction between the screw and nut. Typical values of \(f\) are given in Table 8-5, pg. 414 of the textbook.

Torques for raising and lowering the load for an ACME thread are found by multiplying friction terms of the above equations by \(\frac{1}{\cos \alpha} = \sec \alpha\), and thus

\[
T_R = F \frac{d_m}{2} \left( \frac{\pi f d_m \sec \alpha + l}{\pi d_m - fl \sec \alpha} \right) \quad \text{and} \quad T_L = F \frac{d_m}{2} \left( \frac{\pi f d_m \sec \alpha - l}{\pi d_m + fl \sec \alpha} \right).
\]

For both the square and ACME threads, if there is a collar attached to the screw for the load to sit on, then the friction on this collar should also be included for finding the torques of raising and lowering the load. We now have the total torque \(T\) for raising the load

\[
T = T_R + T_c = T_R + \frac{F f_c d_c}{2}
\]

where \(f_c\) is the coefficient of friction between the collar and the load, and \(d_c\) is the diameter of friction circle for the collar. Typical values of \(f_c\) are given in Table 8-6, pg. 414 of the textbook. The total torque \(T\) for lowering the load is also given by

\[
T = T_L + T_c.
\]

A power screw is said to be self-locking if the nut does not roll back by itself after the load is raised. For the self-locking of a screw, we should have the total torque for lowering the load to be positive, i.e.

\[
T \text{ (for lowering the load)} > 0.
\]
We also define an efficiency $e$ for raising the load

$$e = \frac{T_0}{T}$$

where $T_0$ is the torque for raising the load without the friction, i.e. $T_0 = T_R \big|_{f=0, f_c=0} = \frac{Fl}{2\pi}$, and hence

$$e = \frac{Fl}{2\pi T}.$$

The stresses on a power screw during raising or lowering a load are 3-dimensional. These stresses are given for a square thread as:

1) Axial compressive stress due to the compressive force $F$: $\sigma_y = \frac{4F}{\pi d_r^2}$,

2) Torsional shear stress due to the torque $T$: $\tau_{yz} = \frac{16T}{\pi d_r^2}$, and

3) Bending stress due to bending of thread by $F$: $\sigma_x = \frac{6F}{\pi d_r n_t p}$, where $n_t$ is the total number of threads between the nut and screw, which is usually 3. This bending stress can also be approximated as $\sigma_x = \frac{6(0.38F)}{\pi d_r p}$, which assumes that only the 1st thread between the nut and screw takes the 38% of the load, in which case $n_t = 1$.

Hence, we can now compute the equivalent von Mises stress $\sigma'$ from the above 3 stresses using the equation (5-14) in Chapter 5, pg. 237, as
\[ \sigma' = \frac{1}{\sqrt{2}} \left[ \sigma_x^2 + \sigma_y^2 + \left( \sigma_y - \sigma_x \right)^2 + 6 \tau_{xy}^2 \right]^{1/2} \]

This equivalent stress \( \sigma' \) can be used to find a factor of safety of \( n \) for the power screw to decide on its strength, that is \( n = \frac{S_y}{\sigma'} > 1 \). There is another stress, bearing stress \( \sigma_B \), that is used to judge if the force \( F \) is crushing or damaging the surface of the thread. This stress is calculated to be

\[
\sigma_B = -\frac{F}{\pi d_m n_1 p/2} = -\frac{2F}{\pi d_m n_1 p}, \quad \text{or} \quad \sigma_B = -\frac{2(0.38F)}{\pi d_m p},
\]

assuming again that only the 1st engaged thread between the nut and screw takes the 38% of the load, and hence \( n_1 = 1 \). This bearing stress \( \sigma_B \), must be checked against the allowable bearing pressure values in Table 8-4, pg. 413.

**Note:** Review Example 8-1, pgs. 411-413.

### 8-3 Threaded Fasteners
Various hexagonal-head bolts and cap screws are shown in Figures 8-9 and 8-10, pg. 415, in the textbook. Several machine screw head types are illustrated in Figure 8-11, pg. 416. Dimensions of hexagonal bolts and cap screws are given in the appendix of the textbook, Tables A-29 and A-30, pgs. 1061 and 1062. Hexagonal nuts are shown in Figure 8-12, pg. 416, and their dimensions are provided in the appendix, Table A-31, pg. 1063.

### 8-4 Modeling of Screw Joints – Fastener (Bolt or Screw) Stiffness

A screw joint or connection is idealized through parallel springs with a bolt stiffness (spring constant) of \( k_b \) and member stiffness of \( k_m \) as shown in the figure below:

![Diagram](image)

The bolt stiffness \( k_b \) is found using the length of the screw within the grip of the connection. The model of the bolt is thought as two springs in series, one spring corresponding to the unthreaded portion and the other spring for the threaded portion of the bolt. So, we have

\[
\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t}
\]

where \( k_d \) and \( k_t \) are the stiffnesses of the unthreaded and threaded portions of the screw, which are defined as

\[
k_d = \frac{A_d E}{l_d} \quad \text{and} \quad k_t = \frac{A_t E}{l_t}
\]
where
\[ A_d = \text{cross-sectional area of unthreaded portion} = \frac{\pi}{4} d^2 \] (\(d\) is major diameter),
\[ l_d = \text{length of unthreaded portion}, \]
\[ A_t = \text{tensile stress area found in Tables 8-1 and 8-2 (pgs. 404 and 405), and} \]
\[ l_t = \text{length of threaded portion of the screw within the grip}. \]
Hence, we have from the above equations:
\[ k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}. \]

**8-5 Modeling of Screw Joints – Member Stiffness**

The calculation of the stiffness \(k_m\) for the members clamped by a bolt is not as straightforward. Members are clamped in series, hence we have
\[ \frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots \]
Stiffness for each member in the above equation (\(k_1, k_2, \text{and so on}\)) can be found from
\[ k = \frac{0.5774 \pi E d}{ln \frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}} \]
where \(t\) is the thickness of the member, \(d\) is the major bolt diameter and \(D\) is the bottom side length of the pressure cone as shown in Figure 8-15(b), pg. 420. If there is a soft gasket among the members, then we can set as \(k_m = k_g\), where \(k_g\) is the gasket stiffness. If there are only two clamped members that are identical, i.e. they have the same thickness of \(t\) and are made up of the same material, then the resultant member stiffness is obtained directly from
\[ k_m = \frac{0.5774 \pi E d}{2 ln \left( \frac{5(0.5774t + 0.5d)}{(0.5774t + 2.5d)} \right)} \]
where \(l=2t\). In the case of two identical members we can also use the equation
\[ \frac{k_m}{Ed} = A e^{Bd / l} \]
where the constants \(A\) and \(B\) can be obtained from Table 8-8, pg. 430, in the textbook.

**Note:** Review Example 8-2, pgs. 422-424.

**8-6 Bolt Strength – Material Properties for Bolts**

A bolt strength is specified by its proof and tensile strengths, which are listed in Tables 8-9, 8-10 (UN bolts) and 8-11 (M bolts), pgs. 425 through 427 in the textbook.
A proof load is the maximum load or force that a bolt can withstand before getting into plastic region. Then the proof strength is defined as the proof load divided by the bolt’s tensile stress area.

8-7 and 8-9 Static Analysis of Bolts
The scenario is as follows. We first apply a preload of $F_i$ to the bolt, which subjects the bolt to tension and clamped members to compression. Afterwards, the joint is put into service, where both the bolt and members are subjected to an external load of $P$. This $P$ is taken by the bolt as $P_b$ and by the members as $P_m$. Remember from the previous Section 8-4 that the bolt and members are connected as two parallel springs of $k_b$ and $k_m$. We can find the following relations

$$P_b = \frac{k_b}{k_b + k_m} P = CP$$ and $$P_m = \frac{k_m}{k_b + k_m} P = (1 - C)P$$

where $C$ is called the joint constant. Consequently, we have

$$F_b = \text{resultant bolt load} = P_b + F_i = CP + F_i$$

and

$$F_m = \text{resultant member load} = P_m - F_i = (1 - C)P - F_i$$

The members should always be compressed and hence we always need $F_m < 0$. The preload $F_i$ in the above equations is assumed as

$$F_i = \begin{cases} 0.75 A_t S_p & \text{for reused bolts or screws} \\ 0.90 A_t S_p & \text{for permanent bolts or screws} \end{cases}$$

Now we can calculate the bolt stress as

$$\sigma_b = \frac{F_b}{A_t} = \frac{CP}{A_t} + \frac{F_i}{A_t}.$$ 

Hence, the yield factor of safety of $n_p > 1$ guarding against the static stress exceeding the proof strength is

$$n_p = \frac{S_p}{\sigma_b} = \frac{S_p}{(CP + F_i)/A_t} = \frac{S_p A_t}{CP + F_i}. $$

Multiplying the external load $P$ by a load factor of $n_L$ and letting the bolt stress equal to its proof strength at the limit, we get

$$\frac{C n_L P}{A_t} + \frac{F_i}{A_t} = S_p$$

which we can solve it for the load factor as
\[ n_L = \frac{S_P A_i - F_i}{CP} \].

Remember that always \( n_L > 1 \) and it stands for a kind of factor of safety for the bolt overloading by \( P \). We should also check members against the joint separation (\( n_0 > 1 \)). We have for the members at the limit of separation

\[ (1-C) n_0 P - F_i = 0 \]

from which we obtain

\[ n_0 = \frac{F_i}{(1-C)P} \).

Note: Review Example 8-4, pgs. 434 and 435.

8-8 Required Bolt Torque

We can express the torque \( T \) required to produce a preload of \( F_i \) by

\[ T = K F_i d \]

where \( K \) is obtained for different bolt conditions from Table 8-15, pg. 431. In the absence of any information we can assume that \( K = 0.2 \).

Note: Review Example 8-3 (parts a and b), pgs. 431 and 432.

8-11 Fatigue Analysis

For the fatigue, we assume that the external load changes between \( P_{min} \) and \( P_{max} \). Hence we have the maximum and minimum bolt forces as

\[ (F_b)_{max} = CP_{max} + F_i \quad \text{and} \quad (F_b)_{min} = CP_{min} + F_i \]

Then the maximum and minimum bolt stresses are found as

\[ (\sigma_b)_{max} = \frac{CP_{max}}{A_t} + \frac{F_i}{A_t} = \frac{CP_{max}}{A_t} + \sigma_i \quad \text{and} \quad (\sigma_b)_{min} = \frac{CP_{min}}{A_t} + \sigma_i \]

Thus, we have the alternating and midrange stresses in the bolt as

\[ \sigma_a = \frac{1}{2} (\sigma_b)_{max} - (\sigma_b)_{min} = \frac{C(P_{max} - P_{min})}{2 A_t} \]

and

\[ \sigma_m = \frac{1}{2} (\sigma_b)_{max} + (\sigma_b)_{min} = \frac{C(P_{max} + P_{min})}{2 A_t} + \sigma_i \]

Now we can use one of the fatigue methods to find the fatigue factor of safety \( n_f \) for the bolt. If we use the Gerber approach
which can be solved for \( n_f \). The endurance limit or endurance strength \( S_e \) in the above equation can be obtained directly from Table 8-17, pg. 445 in the textbook, for bolts and screws with rounded threads. Remember that we get the tensile strength \( S_{ut} \) for bolts and screws from Tables 8-9 through 8-11, pgs. 425-427. Fatigue safety factors for the Goodman and ASME-elliptic can also be found in a similar approach. For the Goodman method in specific, the equation for \( n_f \) is directly given by the equation (8-38), pg. 438, as

\[
\frac{n_f \sigma_a}{S_e} + \left( \frac{n_f \sigma_m}{S_{ut}} \right)^2 = 1
\]

**Important Note:** Expressions of \( n_f \) given by equations equations (8-45), (8-46) and (8-47) on pg. 439 for Goodman, Gerber and ASME methods assume that the external load changes between 0 and \( P \), i.e. \( P_{\text{min}} = 0 \) and \( P_{\text{max}} = P \). So, use them only if this is the case.

**Note:** Review Example 8-5, pgs. 440-443.

### 8-12 Shear Joints

**Shear Joints with Eccentric Loading**

When a group of bolts or pins are subjected to shear, we have to find the resultant shear forces and stresses on the bolts and identify the critical bolt or bolts taking the maximum shear stress. A typical situation is shown below:
There are four bolts or pins at the left side of the beam, which are resisting the reaction force of $V_1$ and the moment of $M_1 = Fl$. Reactions $V_1$ and $M_1$ are put in the centroid of the left bolt group, $G$. We have two types of shear forces in each bolt, one is the primary shear force ($F'_A$, $F'_B$, $F'_C$, and $F'_D$) and the other is the secondary shear force ($F'^*_A$, $F'^*_B$, $F'^*_C$, and $F'^*_D$). The primary shear forces are due to the shear force of $V_1$ and hence are equal to

$$F'_A = F'_B = F'_C = F'_D = \frac{V_1}{n}$$

where $n$ is the number bolts in the bolt group (it is 4 in this case). The secondary shear forces exist to resist the bending moment $M_1$ and thus we have

$$F'^*_A = \frac{M_1 r_A}{r^2_A + r^2_B + r^2_C + r^2_D}$$

and so on.

As shown in the figure, we have to find out the resultant shear force in each bolt ($F_A$, $F_B$, $F_C$, and $F_D$) and divide it by the cross-sectional area to calculate the shear stress. It looks like in the figure that the bolts $B$ and $D$ are taking the maximum shear forces. Hence, assuming that all the bolts are of the same size (diameter), the bolts $B$ and $D$ are the critical ones, because they get the maximum shear stresses.

**Note:** Review Example 8-7, pgs. 448-450.