MECHANICAL VIBRATIONS EXPERIMENT

THE STUDY OF VIBRATIONS

Vibrations are oscillations of a mechanical or structural system about an equilibrium position. Vibrations are initiated when an inertia element is displaced from its equilibrium position due to an energy imparted to the system through an external source. A restoring force, or a conservative force developed in a potential energy element, pulls the element back toward equilibrium. When work is done on the block of Figure 1.1(a) to displace it from its equilibrium position, potential energy is developed in the spring. When the block is released the spring force pulls the block toward equilibrium with the potential energy being converted to kinetic energy. In the absence of non-conservative forces, this transfer of energy is continual, causing the block to oscillate about its equilibrium position. When the pendulum of Figure 1.1(b) is released from a position above its equilibrium position the moment of the gravity force pulls the particle, the pendulum bob, back toward equilibrium with potential energy being converted to kinetic energy. In the absence of non-conservative forces, the pendulum will oscillate about the vertical equilibrium position.

(a) When the block is displaced from equilibrium, the force developed in the spring (as a result of the stored potential energy) pulls the block back toward the equilibrium position. (b) When the pendulum is rotated away from the vertical equilibrium position, the moment of the gravity force about the support pulls the pendulum back toward the equilibrium position.

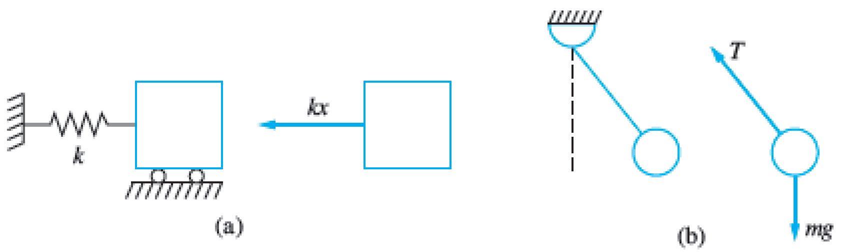


Figure 1.1

Vibrations occur in many mechanical and structural systems. If uncontrolled, vibration can lead to catastrophic situations. Vibrations of machine tools or machine tool chatter can lead to improper machining of parts. Structural failure can occur because of large dynamic stresses developed during earthquakes or even wind-induced vibration. Vibrations induced by an unbalanced helicopter blade while rotating at high speeds can lead to the blade's failure and catastrophe for the helicopter. Excessive vibrations of pumps, compressors, turbo machinery, and other industrial machines can induce vibrations of the surrounding structure, leading to inefficient operation of the machines while the noise produced can cause human discomfort.

Vibrations can be introduced, with beneficial effects, into systems in which they would not naturally occur. Vehicle suspension systems are designed to protect passengers from discomfort when traveling over rough terrain. Vibration isolators are used to protect structures from excessive forces developed in the operation of rotating machinery. Cushioning is used in packaging to protect fragile items from impulsive forces. Energy harvesting takes unwanted vibrations and turns them into stored energy. An energy harvester is a device that is attached to an automobile, a machine, or any system that is undergoing vibrations. The energy harvester has a seismic mass which vibrates when excited, and that energy is captured electronically.

The Tacoma Narrows Bridge collapsed due to wind induced resonance on November 7th, 1940. Resonance is a process in which an object's, in this case a bridge's, natural vibrating frequency is amplified by an identical frequency. In this case the identical frequency was caused by strong wind gusts blowing across the bridge, creating regions of high and low pressure above and below the bridge (Bernoulli’s principle). This produced violent oscillations, or waves, in the bridge leading to its collapse. In layman's terms, the wind was forced either above or below the bridge, causing the bridge to be moved up or down. This tensed or relaxed the supporting cables, which acted much like rubber bands, and increased the waves in the bridge. These waves were so intense that a person driving across the bridge often lost sight of the car ahead as it dropped into a trough, low point, of the wave.

The following pictures show the violent twisting waves that the bridge withstood prior to its collapse.

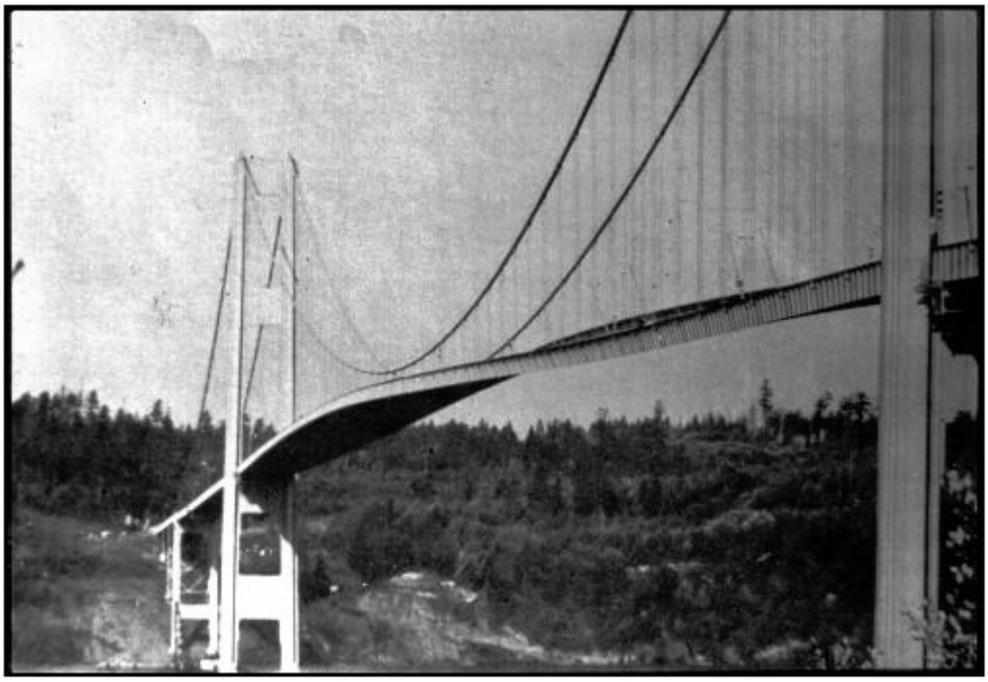


Figure 1.2

Importance of the Study of Vibration

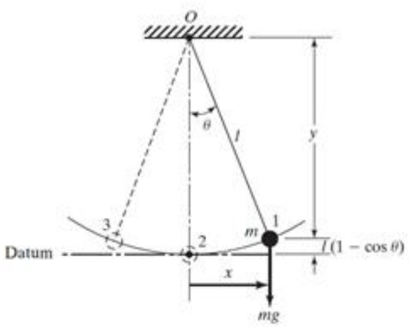
- Vibrations can lead to excessive deflections and failure on the machines and structures
- To reduce vibration through proper design of machines and their mountings
- To utilize profitably in several consumer and industrial applications
- To improve the efficiency of certain machining, casting, forging & welding processes
- To stimulate earthquakes for geological research and conduct studies in design of nuclear reactors

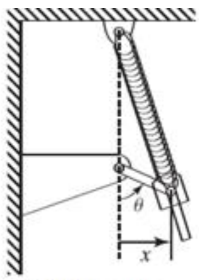
- Vibratory System basically consists of:
  - spring or elasticity
  - mass or inertia
  - damper
- Vibration Involves transfer of potential energy to kinetic energy and vice versa

Degree of Freedom *(d.o.f.)*:

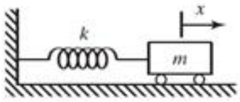
Minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time

- Examples of single degree-of-freedom systems:

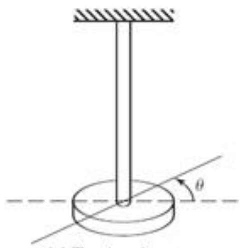




(a) Slider-crank-spring mechanism

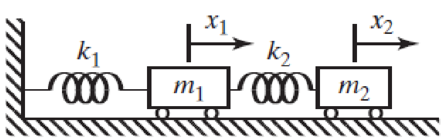


(b) Spring-mass system

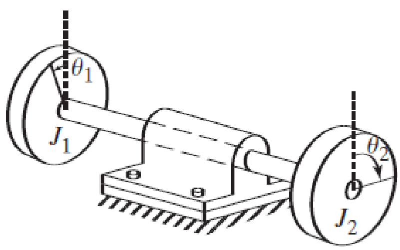


(c) Torsional system

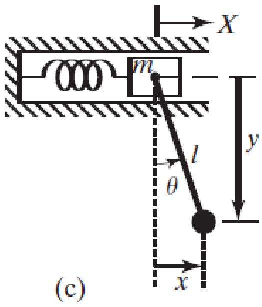
- Examples of second degree-of-freedom systems:



(a)

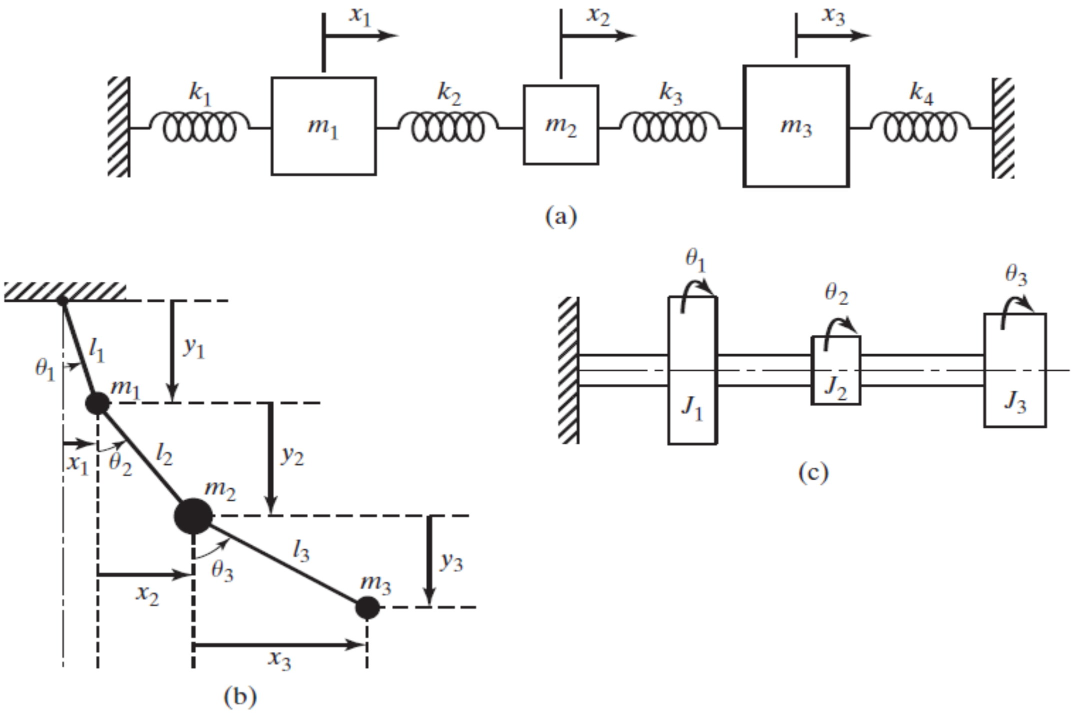


(b)

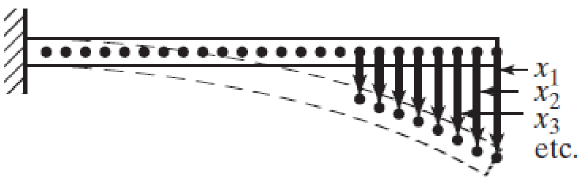


(c)

- Examples of three degree-of-freedom systems:



- Example of Infinite-number-of-degrees-of-freedom system:



- *Infinite* number of degrees of freedom system are termed *continuous* or *distributed* systems
- *Finite* number of degrees of freedom are termed *discrete* or *lumped* parameter systems
- More accurate results obtained by increasing number of degrees of freedom

Free Vibration:

A system is left to vibrate on its own after an initial disturbance and no external force acts on the system. E.g. simple pendulum

Forced Vibration:

A system that is subjected to a repeating external force. E.g. oscillation arises from diesel engines**.**

Resonance:

It occurs when the frequency of the external force coincides with one of the natural frequencies of the system

Undamped Vibration:

When ***no*** energy is lost or dissipated in friction or other resistance during oscillations

Damped Vibration:

When ***any*** energy is lost or dissipated in friction or other resistance during oscillations

Linear Vibration:

When ***all*** basic components of a vibratory system, i.e. the spring, the mass and the damper behave linearly

Nonlinear Vibration:

If ***any*** of the components behave nonlinearly

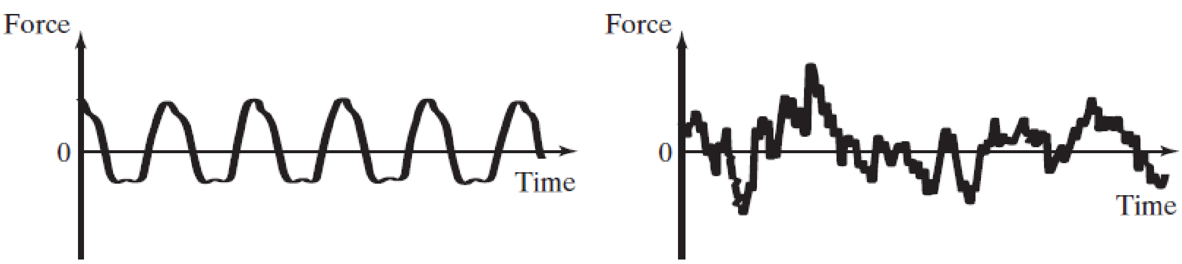
Deterministic Vibration:

If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time

Nondeterministic or random Vibration:

When the value of the excitation at a given time cannot be predicted

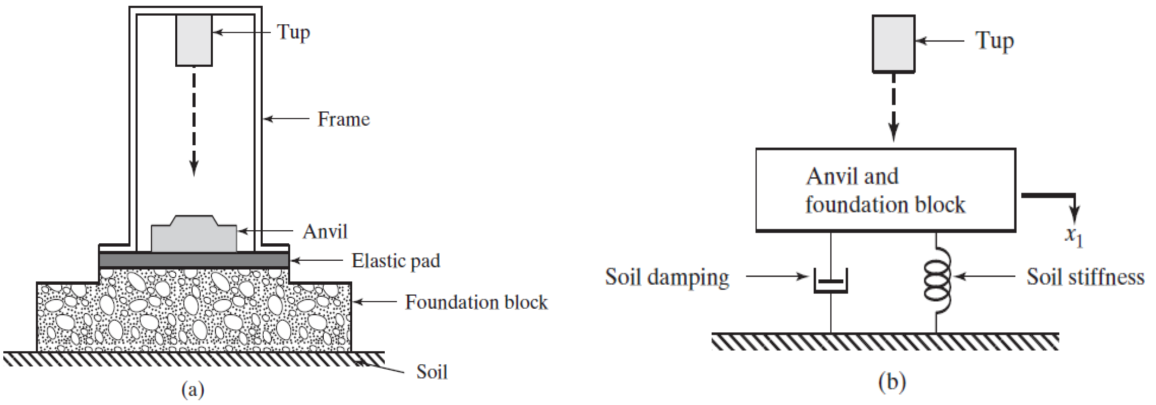
- Examples of deterministic and random excitation:

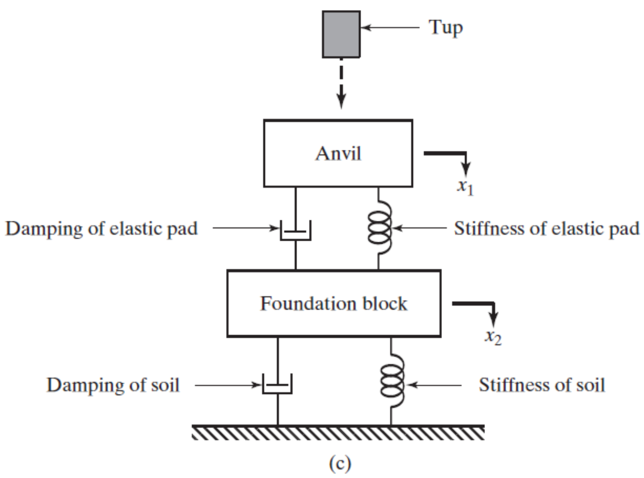


(a) A deterministic (periodic) excitation (b) A random excitation

Modeling of the mechanical systems:

Example: a forging hammer





Spring Elements:

- *Linear* spring is a type of mechanical link that is generally assumed to have negligible mass and damping
- *Spring force* is given by:

$$F = kx$$

F = spring force
k = spring stiffness or spring constant
x = deformation (displacement of one end with respect to the other)

- *Static deflection* of a beam at the free end is given by:

$$\delta_{st} = \frac{Wl^3}{3EI}$$

W = mg is the weight of the mass m,
E = Young's Modulus, and
I = moment of inertia of cross-section of beam

- *Spring Constant* is given by:

$$k = \frac{W}{\delta_{st}} = \frac{3EI}{l^3}$$

- Combination of Springs:

1) Springs in parallel – if we have *n* spring constants k_1, k_2, …, k_n in *parallel*, then the equivalent spring constant k_{eq} is:

$$k_{eq} = k_1 + k_2 + \ldots + k_n$$

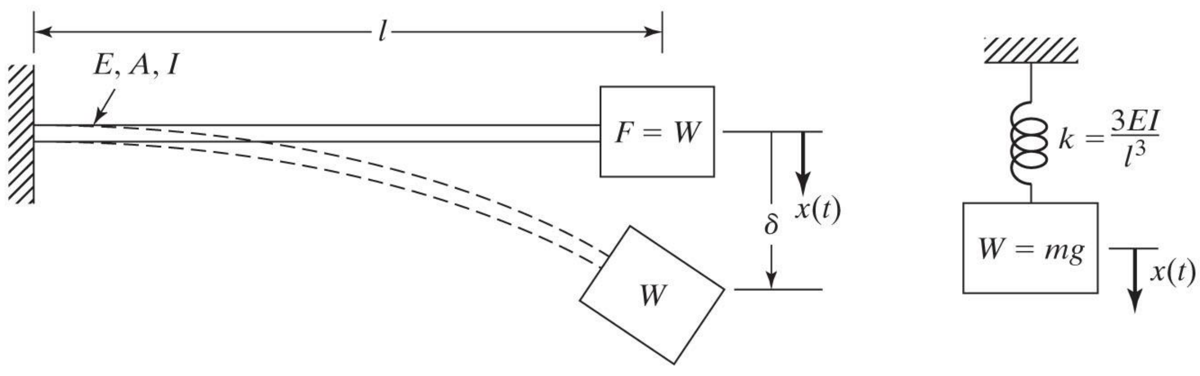
2) Springs in series – if we have *n* spring constants k_1, k_2, …, k_n in *series*, then the equivalent spring constant k_{eq} is:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots + \frac{1}{k_n}$$

Mass or Inertia Elements:

- Using mathematical model to represent the actual vibrating system

  E.g. In the figure below, the mass and damping of the beam can be disregarded; the system can thus be modeled as a spring-mass system as shown.



(a) Cantilever with end force (b) Equivalent spring

Damping Elements:

- **Viscous Damping:**

  Damping force is proportional to the velocity of the vibrating body in a fluid medium such as air, water, gas, and oil.

- **Coulomb or Dry Friction Damping:**

  Damping force is constant in magnitude but opposite in direction to that of the motion of the vibrating body between dry surfaces

- **Material or Solid or Hysteretic Damping:**

  Energy is absorbed or dissipated by material during deformation due to friction between internal planes

Harmonic Motion:

- Periodic Motion: motion repeated after equal intervals of time
- Harmonic Motion: simplest type of periodic motion
- Displacement (x): *(on horizontal axis)*

$$x = A\sin\theta = A\sin\omega t$$

- Velocity:

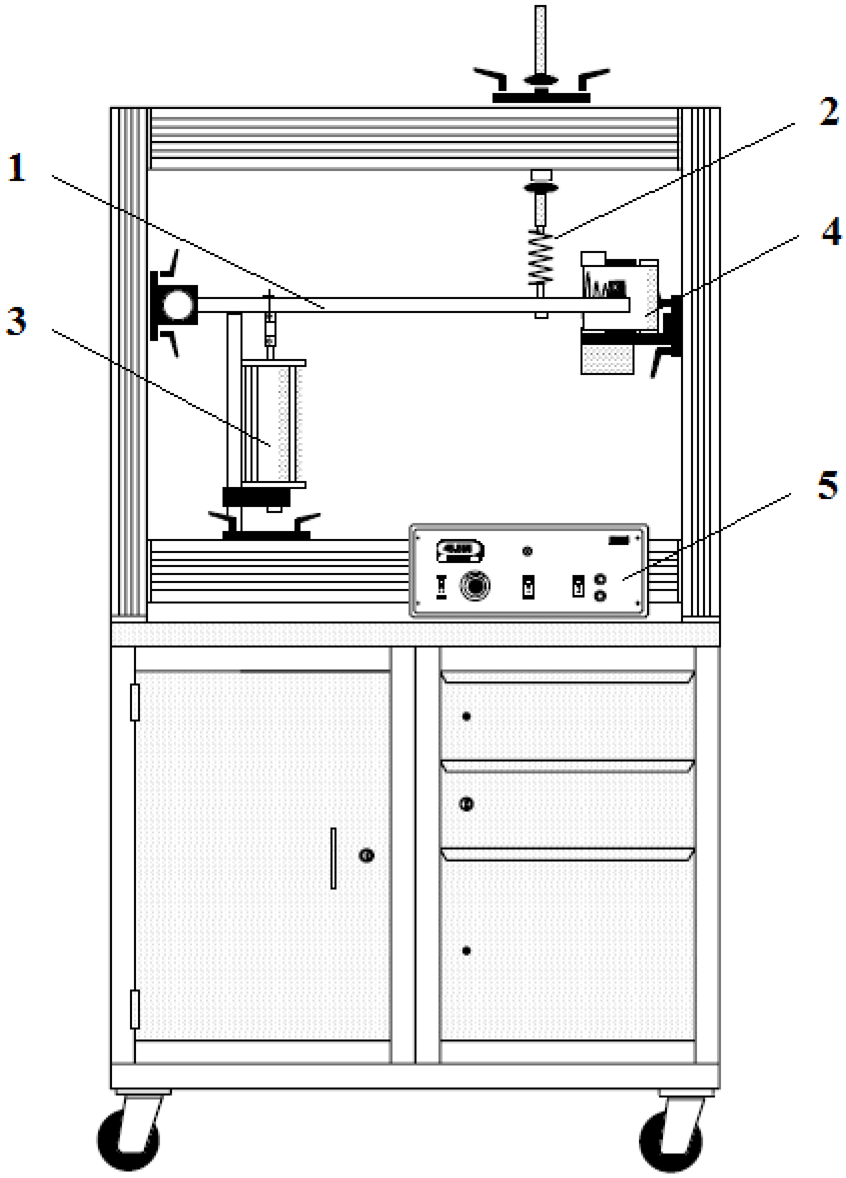
$$\frac{dx}{dt} = \omega A\cos\ \omega t$$

- Acceleration:

$$\frac{d^2x}{dt^2} = -\omega^2 A\sin\ \omega t = -\omega^2 x$$

Undamped Oscillation Experiment

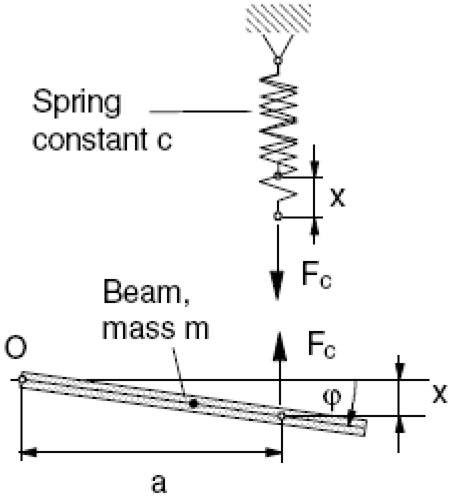
The Oscillation Training System is housed on a laboratory trolley. As you can see in the figure below, 1 shows the cantilever beam which has a free support that allows to rotation of the beam. Member 2 is representing the spring element and its applying place and stiffness can be changed. Member 3 is damping element that is not used in this experiment. 4th member has a duty of recording the frequency with a pen on it. Finally, member 5 represents the control unit.



Aim of the experiment:

This experiment is designed to observe the change of the natural frequency due to the change of lever arm length. Experimental and calculated natural frequencies will also be compared with each other.

Equation of Motion:



After mathematically modelling the system, equation of motion of the vibration is obtained using Newton's laws or Energy method. Positive direction is CCW in this system. First, displacement of the spring should be established.

$$x = a\,sin\theta$$

and for small amplitudes, it can be accepted that:

$$sin\theta = \theta, \qquad cos\theta = 1$$

Establishment of the equation of motion involves forming the moment equilibrium about the fulcrum point O of the beam.:

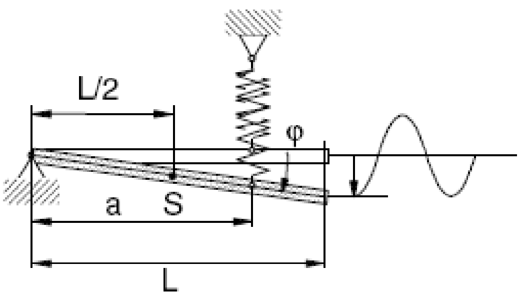
$$\Sigma M_o = I_o \ddot{\varphi} = -F_c a$$

Here, *mg* weight of the beam is not taken into consideration because of measuring the x at equilibrium position. The spring force Fc results from the deflection *x* and the spring constant *k*. For a small angle, the deflection can be formed from torsion φ and lever arm *a*

$$F_c = kx = k\,\varphi\,a$$

The mass moment inertia of the beam about the fulcrum point is

$$I_o = \frac{mL^2}{3}$$



The equation of motion is thus the following homogeneous differential equation

$$\ddot{\varphi} + \frac{3ka^2}{mL^2}\varphi = 0$$

The solution produces harmonic oscillations with the natural angular frequency ω_0 or the natural frequency *f*

$$w_n^2 = \frac{3ka^2}{mL^2}, \quad f = \frac{1}{2\pi}\sqrt{\frac{3ka^2}{mL^2}}$$

$$w_n T = 2\pi$$

The periodic time is

$$T = 2\pi\sqrt{\frac{mL^2}{3ka^2}}$$

As can be seen, the periodic time/natural frequency can easily be set by way of the lever arm *a* of the spring. The natural frequency of the undamped free vibration is:

$$w_n = \sqrt{\frac{k}{m}}$$

Performing Steps of the Experiment:

- Mount spring accordingly and secure with lock nuts
- Horizontally align beam
- Insert pen
- Start plotter
- Deflect beam by hand and let it oscillate
- Stop plotter

Repeat experiment with other springs and lever arms

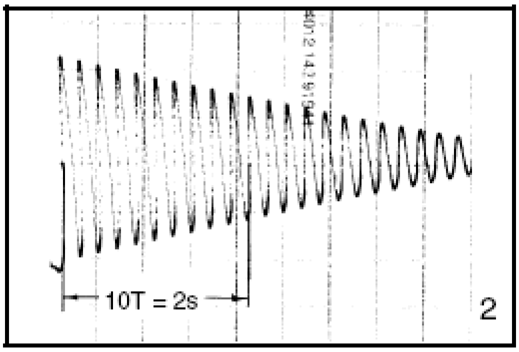
Mass of beam m = 1.680 kg

Length of beam L = 732 mm

Testing involves the following combinations:

Influence of spring constant Calculated frequences			
Experiment	Spring no., constant c in N/mm	Lever arm a in mm	Natural frequency f in Hz
1	1, 0.75	350	2.78
2	1, 0.75	650	5.17
3	2, 1.50	350	3.94
4	2, 1.50	650	7.31
5	3, 3.00	650	10.34

Result of Experiment 2:



REPORT: Please prepare your report in pdf format and deliver it to mcyilmaz@ybu.edu.tr in one week. Your report should have the followings;

a) Cover (with names and numbers) (1 page)
b) A short introduction (1 page)
c) All the necessary calculations using measured data.
   1. Calculation of stiffness of the spring
   2. Calculation of natural frequencies using formula
   3. Comparing the frequencies with the values that obtained from graphics.
   4. Comparing the frequencies with the values from table and calculation of the error rate.
d) Discussion of your results and a conclusion (1/2 page).