FACULTY OF ENGINEERING AND NATURAL SCIENCES

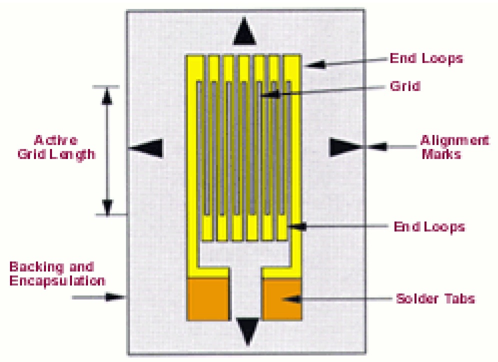
DEPARTMENT

OF

MECHANICAL ENGINEERING

MCE 403 MACHINERY LABORATORY

EXPERIMENT 10: STRAIN MEASUREMENT BY USING STRAIN GAUGE



1.OBJECTIVE

The objective of this experiment is to become familiar with the electric resistance strain gauge techniques and utilize such gauges for the determination of unknown quantities (such as strain, stress and young's modulus) at the prescribed conditions of a cantilever beam.

2.INTRODUCTION

Experimental stress analysis is an important tool in the design and testing of many products. Several practical techniques are available including photoelastic, coatings and models, brittle coatings, and electrical resistance strain gauges.

In this experiment, the electrical resistance strain gauge will be utilized. There are three steps in obtaining experimental strain measurements by using a strain gauge:

1. Selecting a strain gauge
2. Mounting the gauge on the test structure
3. Measuring strains corresponding to specific loads.

The operation and selection criteria for strain gauges will be discussed. In this experiment, you will mount a strain gauge on a beam and test its accuracy. Measurements will be made with a strain gauge rosette in this experiment to obtain the principal stresses and strains on a cantilevered beam.

What's a Strain Gauge Used For?

The Birdman Contest is an annual event held on Lake Biwa near Kyoto, Japan. In this contest cleverly designed human-powered airplanes and gliders fly several hundred meters across the lake. Aside from the great spectacle of this event, it is a wonderful view of engineering experimentation and competition. Despite the careful designs and well-balanced airframes occasionally the wings of these vehicles fail and crash into the lake. There have been some spectacular crashes but few, if any, injuries to the contestants.

Increasingly, each time a new airplane, automobile, or other vehicle is introduced, the structure of such vehicles is designed to be lighter to attain faster running speeds and less fuel consumption. It is possible to design a lighter and more efficient product by selecting light-weight materials. However, as with all technology, there are plusses and minuses to be balanced. If a structural material is made lighter or thinner the safety of the vehicle is compromised unless the required strength is maintained. By the same token, if only the strength is taken into consideration, the vehicle's weight will increase and its economic feasibility is compromised.

In engineering design the balance between safety and economics is one variable in the equation of creating a successful product. While attempting to design a component or vehicle that provides the appropriate strength it is important to understand the stress borne by the various parts under different conditions. However, there is no technology or test tool that allows **direct** measurement of stress. Thus, strain on the surface is frequently measured in

order to determine internal stress. Strain gauges are the most common instrument to measure surface strain.	

STRAIN GAUGES:

There are many types of strain gauges. The fundamental structure of a strain gauge consists of a grid-shaped sensing element of thin metallic resistive foil (3 to 6 microns thick) that is sandwiched between a base of thin plastic film (12-16 micron thick) and a covering or lamination of thin film.

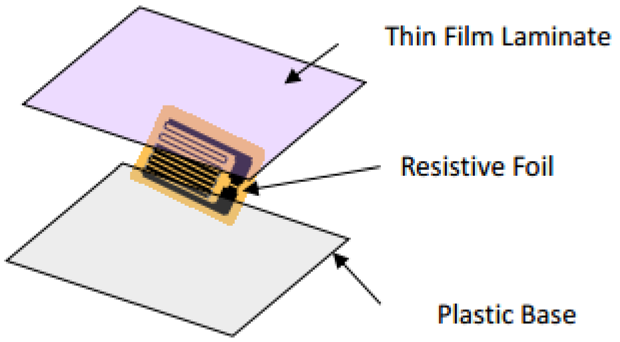


Figure 1. Strain gauge construction

STRAIN GAUGE OPERATION:

Strain gauge is tightly bonded to the specimen. Therefore, depending that unit deformation on the specimen, the sensing element may elongate or contract. During elongation or contraction, electrical resistance of the metal wire changes. The strain gauge measure the strain on the specimen by means of the principle resistance changes. Generally, sensing element are made of copper-nickel alloy in strain gauge. Depending the strain on the alloy plate, the resistance changes at a fix rate.

$$\frac{\Delta R}{R} = Ks.\varepsilon \qquad (1)$$

R: The initial resistance of the strain gauge, Ω (ohm)
ΔR: The change of the resistance, Ω (ohm)
Ks: Gauge Factor, Proportional constant
ε : Strain

Gauge factor, Ks, changes according to the material being used in strain gauge. Generally, Gauge factor of copper-nickel alloy strain gauges is approximately 2 or 2.1. Strain gauges, generally have 120 or 350 Ω resistance.
It is very difficult to accurately measure such a small resistance change, and also, it is not possible to use an ohmmeter to measure. Thus, Wheatstone bridge electric circuit are used to measure the resistance changes.

3. THEORY

STRESS:

Stress is simply a distributed force on an external or internal surface of a body. To obtain a physical feeling of this idea, consider being submerged in water at a particular depth. The ‘‘force’’ of the water one feels at this depth is a pressure, which is a compressive stress, and not a finite number of ‘‘concentrated’’ forces. Other types of force distributions (stress) can occur in a liquid or solid. Tensile (pulling rather than pushing) and shear (rubbing or sliding) force distributions can also exist.

Consider a general solid body loaded as shown in Figure 2 (a). Pi and pi are applied concentrated forces and applied surface force distributions, respectively; and R_i and r_i are possible support reaction force and surface force distributions, respectively. To determine the state of stress at point Q in the body, it is necessary to expose a surface containing the point Q. This is done by making a planar slice, or break, through the body intersecting the point Q. The orientation of this slice is arbitrary, but it is generally made in a convenient plane where the state of stress can be determined easily or where certain geometric relations can be utilized. The first slice, illustrated in Figure 2 (b), is described by the surface normal oriented along the x axis. This establishes the yz plane. The external forces on the remaining body are shown, as well as the internal force (stress) distribution across the exposed internal surface containing Q. In the general case, this distribution will not be uniform along the surface, and will be neither normal nor tangential to the surface at Q. However, the force distribution at Q will have components in the normal and tangential directions. These components will be tensile or compressive and shear stresses, respectively.

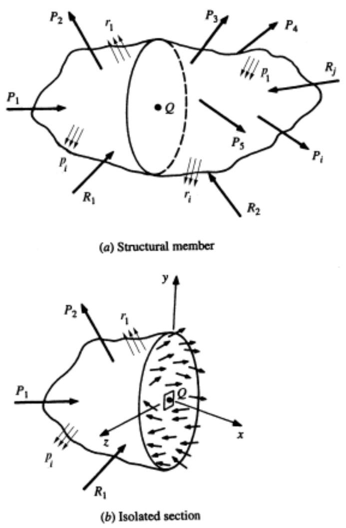


Figure 2. (a) Structural member and (b) Isolated section

Following a right-handed rectangular coordinate system, the y and z axes are defined perpendicular to x, and tangential to the surface. Examine an infinitesimal area $\Delta A_x = \Delta y \Delta z$ surrounding Q, as shown in Figure. 3 (a). The equivalent concentrated force due to the force distribution across this area is ΔF_x, which in general is neither normal nor tangential to the surface (the subscript x is used to designate the normal to the area). The force ΔF_x has components in the x, y, and z directions, which are labeled ΔF_{xx}, ΔF_{xy}, and ΔF_{xz}, respectively, as shown in Figure 3 (b). Note that the first subscript denotes the direction normal to the surface and the second gives the actual direction of the force component. The average distributed force per unit area (average stress) in the x direction is

$$\bar{\sigma}_{xx} = \frac{\Delta F_{xx}}{\Delta A_X} \tag{2}$$

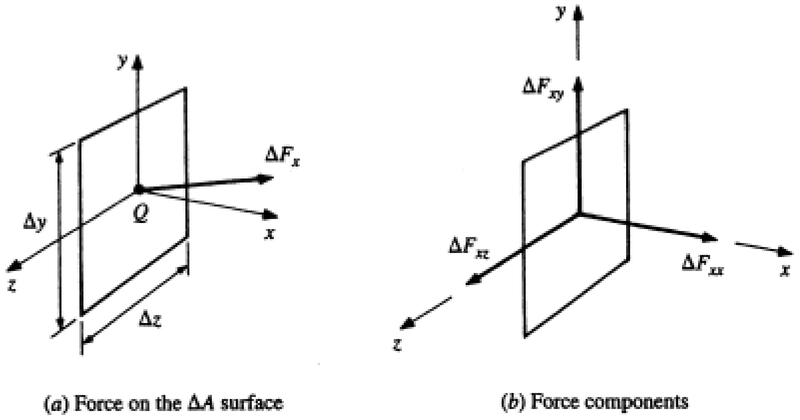


Figure 3. (a) Force on the ΔA surface, (b) Force components

Recalling that stress is actually a point function, we obtain the exact stress in the x direction at point Q by allowing ΔA_x to approach zero. Thus,

$$\sigma_{xx} = \lim_{\Delta A_x \to \infty} \frac{\Delta F_{xx}}{\Delta A_X} \tag{3}$$

or,

$$\sigma_{xx} = \frac{dF_{xx}}{dA_X} \tag{4}$$

STRAIN:

As with stresses, two types of strains exist: normal and shear strains, which are denoted by ε and γ, respectively. Normal strain is the rate of change of the length of the stressed element in a particular direction. Let us first consider a bar with a constant cross-sectional area which has the undeformed length *l*. Under the action of tensile forces (Figure 4) it gets slightly longer. The elongation is denoted by Δ*l* and is assumed to be much smaller than the original length *l*. As a measure of the amount of deformation, it is useful to introduce, in addition to the elongation, the ratio between the elongation and the original (undeformed) length:

$$\varepsilon = \frac{\Delta l}{l} \tag{5}$$

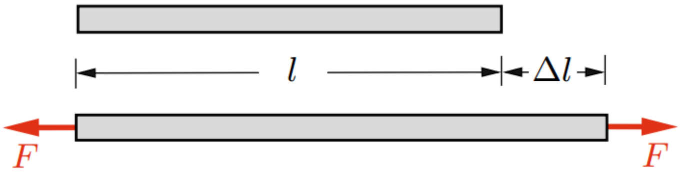


Figure 4. The undeformed length *l* and the deformed length *l*

The dimensionless quantity ε is called strain.

HOOK'S LAW

The strains in a structural member depend on the external loading and therefore on the stresses. For linear elastic behavior, the relation between stresses and strains is given by Hooke's law. In the uniaxial case (bar) it takes the form $\sigma = E\,\varepsilon$ where E is Young's modulus.

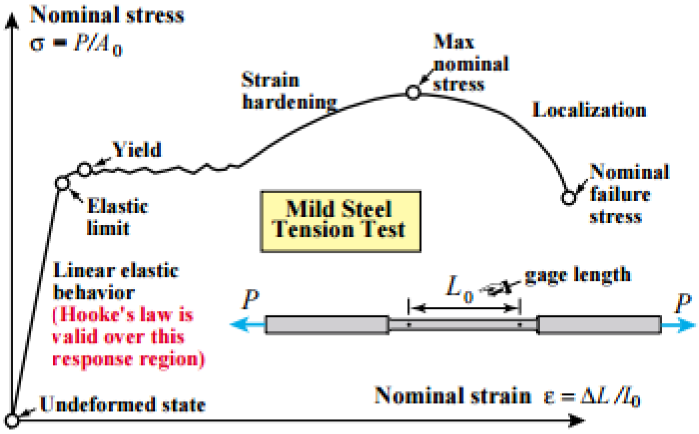


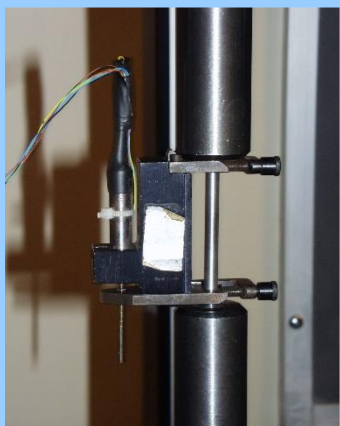
Figure 5. Stress vs strain diagram

Strain Measurement

It should be noted that there are various types of strain measuring methods available. These may be roughly classified into mechanical, electrical, and even optical techniques.

From a geometric perspective, strain recorded during any test may be regarded as a distance change between two points on a test article. Thus all techniques are simply a way of measuring this change in distance.

If the elastic modulus of the test article's constituent material is known, strain measurement will allow calculation of stress. As you have learned from your studies and prior labs strain measurement is often performed to determine the stress created in a test article by some external force, rather than to simply gain knowledge of the strain value itself.



This **linear variable differential transformer** (LVDT), attached to a tensile specimen, is also a common tool for measuring strain.

WHEATSTONE BRIDGE:

Wheatstone bridge is an electric circuit that is used for measuring the instantaneous change in the instant resistance.

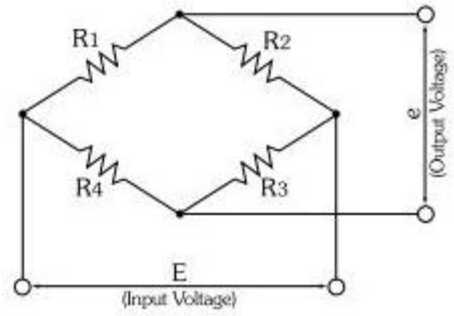


Figure 6. Wheatstone Bridge

$$R1 = R2 = R3 = R3 \quad (6)$$

or

$$R1XR3 = R2XR4 \quad (7)$$

When applying any voltage to input, the output of the system may be zero “0”. In this way, the bridge is in balance. When the any resistance changes, the output will be different than zero.

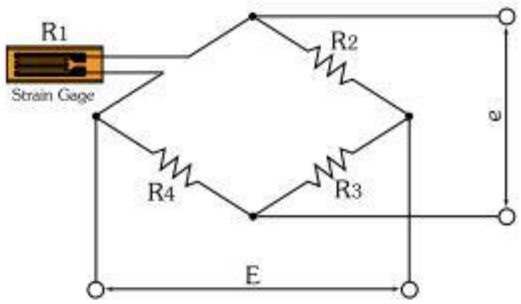


Figure 7. Quarter Wheatstone Bridge

A strain gauge connects to the circuit in Figure 7. When strain gauge loads and the resistance changes, the voltage is obtained at the output of the bridge.

$$e = \frac{1}{4} . \frac{\Delta R1}{R1} . E \quad (7)$$

and

$$e = \frac{1}{4} . Ks. \varepsilon_1 . E \quad (9)$$

Two strain gauged connect to the circuit in Figure 8. When strain gauges load and the resistances change, the voltage is obtained at the output of the bridge.

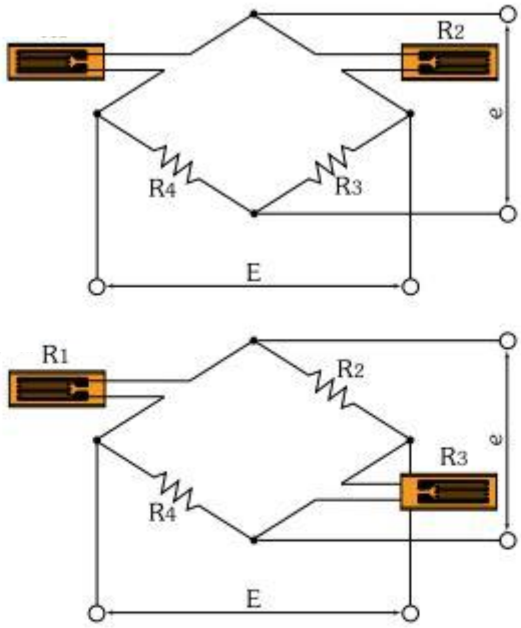


Figure 8. Half Wheatstone Bridge

$$e = \frac{1}{4}.\left(\frac{\Delta R1}{R1} - \frac{\Delta R2}{R2}\right).E \tag{10}$$

and

$$e = \frac{1}{4}.Ks.(\varepsilon_1 - \varepsilon_2).E \tag{11}$$

or

$$e = \frac{1}{4}.\left(\frac{\Delta R1}{R1} + \frac{\Delta R3}{R3}\right).E \tag{12}$$

and

$$e = \frac{1}{4}.Ks.(\varepsilon_1 + \varepsilon_3).E \tag{13}$$

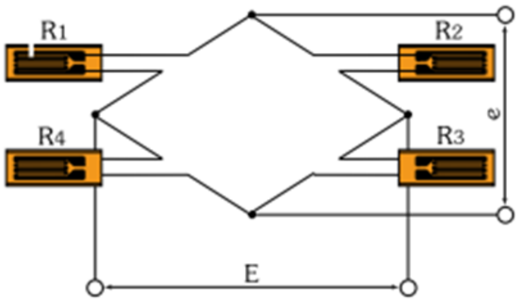


Figure 9. Full Wheatstone Bridge

$$e = \frac{1}{4}.\left(\frac{\Delta R1}{R1} - \frac{\Delta R2}{R2} + \frac{\Delta R3}{R3} - \frac{\Delta R4}{R4}\right).E \tag{14}$$

and

$$e = \frac{1}{4}.Ks.(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4).E \quad (15)$$

A resistance strain gage consists of a thin strain-sensitive wire mounted on a backing that insulates the wire from the test structure. Strain gages are calibrated with a gage factor F, which relates strain to the resistance change in the wire by

$$F = \frac{\Delta R / R}{\Delta L \;/\; L} \quad (16)$$

where R is the resistance and L is the length of the wire. The change in resistance corresponding to typical values of strain is usually only a fraction of an ohm.

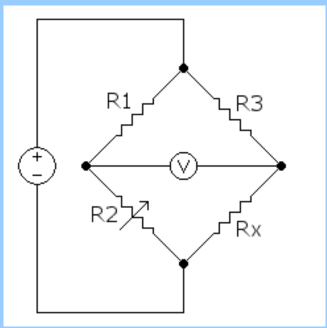
Because conventional ohmmeters are not capable of measuring these small changes in resistance accurately, a Wheatstone bridge is usually employed. It can be operated in either a balanced or unbalanced configuration. For an unbalanced bridge, a change in resistance is measured as a non-zero voltage V_o which, can be calibrated in standard strain units ($\Delta L/L \times 10^{-6}$) or micro strain. A balanced bridge is rebalanced after each load increment so that the output voltage V_o is zero. The appropriate changes in resistance are then noted and strain calculated using the gage factor.

The Wheatstone Bridge

A Wheatstone bridge is a measuring instrument that, despite popular myth, was **not** invented by Sir Charles Wheatstone, but by Samuel H. Christie in 1833. The device was later improved upon and popularized by Wheatstone. The bridge is used to measure an unknown electrical resistance by balancing two legs of a circuit, one leg of which includes the unknown component that is to be measured. The Wheatstone bridge illustrates the concept of a difference measurement, which can be extremely accurate. Variations on the Wheatstone bridge can be used to measure capacitance, inductance, and impedance.

In a typical Wheatstone configuration, Rx is the unknown resistance to be measured; R1, R2 and R3 are resistors of known resistance and the resistance of R2 is adjustable. If the ratio of the two resistances in the known leg (R2/R1) is equal to the ratio of the two in the unknown leg (Rx/R3), then the voltage between the two midpoints will be zero and no current will flow between the midpoints. R2 is varied until this condition is reached. The current direction indicates if R2 is too high or too low.

Detecting zero current can be done to extremely high accuracy. Therefore, if R1, R2 and R3 are known to high precision, then Rx can be measured to high precision. Very small changes in Rx disrupt the balance and are readily detected.



Typical Wheatstone Bridge diagram with strain gauge at Rx

Alternatively, if R1, R2, and R3 are known, but R2 is not adjustable, the voltage or current flow through the meter can be used to calculate the value of Rx. This setup is what you will use in strain gauge measurements, as it is usually faster to read a voltage level off a meter than to adjust a resistance to zero the voltage.	

CANTILEVER BEAM

The beam with the strain gage you have just attached will be placed in the Cantilever Flexure Frame to take strain measurements. The arrangement is schematically shown in Figure 10.

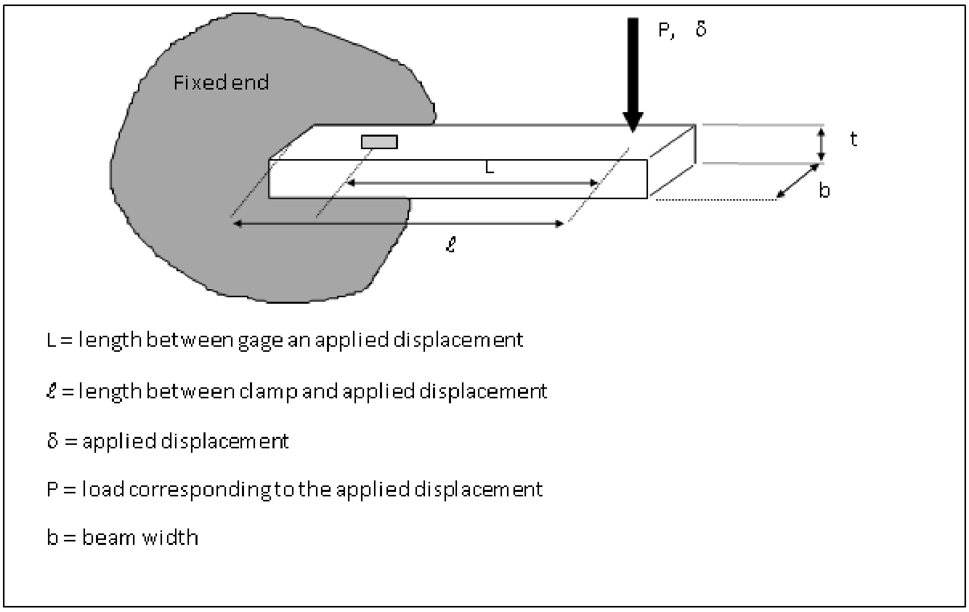


Figure 10. Beam with Strain Gage in Flexure Fixture

The structure examined in this experiment is the cantilever beam. A beam under bending can be characterized by equation (11).

$$\frac{1}{\rho} = \frac{M}{EI} \qquad (17)$$

The radius of curvature is given by equation (12)

$$\frac{1}{\rho} = \frac{d^2y/_{dx^2}}{\left(1 + \left(^{dy}/_{dx}\right)^2\right)^{3/2}} \qquad (18)$$

where y is the deflection in the y direction at any given point x along the beam. Any expression involving the radius of curvature seems to always have it appear in the denominator. And this is no exception, even when it is a defining equation. The fact that many mechanics applications involve bending, but on a small scale. The beam bending discussed here is no exception. In such cases, the best approach is to define the x-axis along the beam such so that the y deflections, and more importantly the deformed slope, y′, will both be small. If y′<<1, then y′ can be neglected in the above equation. It means that, the deflection is very small for many problems. This means that the denominator can be neglected in most cases.

$$\frac{1}{\rho} \approx {d^2y}/{dx^2} \qquad (19)$$

Combining equations (17) and (19) yields.

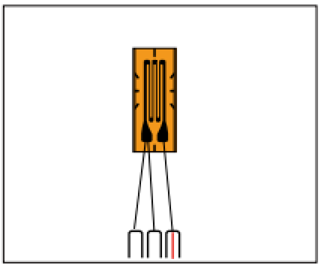
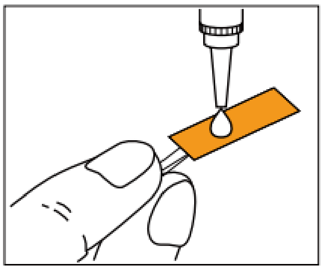
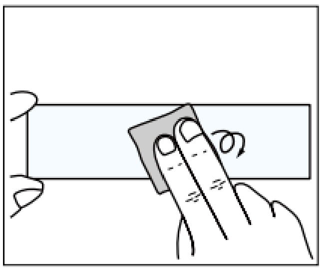
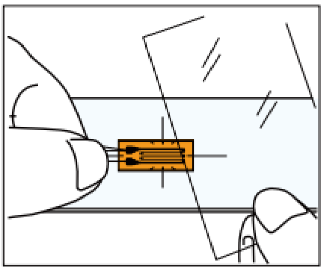
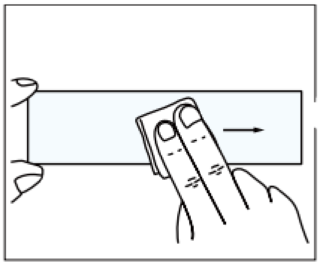
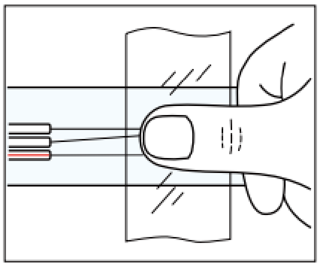
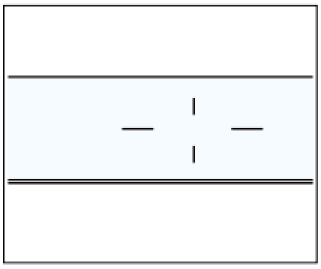
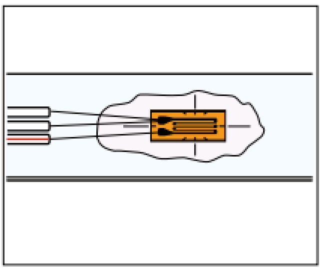
$$\frac{M}{EI} = {d^2y}/{dx^2} \qquad (20)$$

This further reduces to a convenient form of the equation for stress in the cantilever beam.

$$\sigma = \frac{Mc}{I} \qquad (21)$$

Also, there should be some observation about the usability and reliability of the relatively crude instrumentation involved in the experiment. In most cases, strain values differ at most by 5 µstrain from the actual values. In most of the experiments here, that relates to much less than an ounce of resolution. In the laboratory most load cells typically fall within 0.5 % error

4. STRAIN GAUGE BONDING PROCEDURE

(1) Select strain gage.  Select the strain gauge model and gauge length which meet the requirements of the measuring object and purpose	**(5) Apply adhesive.**  Ascertain the back and front of the strain gauge. Apply a drop of adhesive to the back of the strain gauge. Do not spread the adhesive. If spreading occurs, curing is adversely accelerated, thereby lowering the adhesive strength.
(2) Remove dust and paint.  Using a sand cloth (20 to 300), polish the strain gauge bonding site over a wider area than the strain gauge size. Wipe off paint, rust and plating, if any, with a grinder or sand blast before polishing.	**(6) Bond strain gage to measuring site.**  After applying a drop of the adhesive, put the strain gauge on the measuring site while lining up the center marks with the marking off lines.
(3) Remove grease from bonding surface and clean.  Using an industrial tissue paper (SILBON paper) dipped in acetone, clean the strain gauge bonding site. Strongly wipe the surface in a single direction to collect dust and then remove by wiring in the same direction. Reciprocal wiping causes dust to move back and forth and does not ensure cleaning.	**(7) Press strain gage.**  Cover the strain gauge with the accessory polyethylene sheet and press it over the sheet with a thumb. Quickly perform steps (5) to (7) as a series of actions. Once the strain gauge is placed on the bonding site, do not lift it to adjust the position. The adhesive strength will be extremely lowered.
(4) Decide bonding position.  Using a pencil or marking off pin, mark the measuring site in the strain direction. When using a marking off pin, take care not to deeply scratch the strain gauge bonding surface.	**(8) Complete bonding work.**  After pressing the strain gauge with a thumb for one minute or so, remove the polyethylene sheet and make sure the strain gauge is securely bonded. The above steps complete the bonding work. However, good measurement results are available after 60 minutes of complete curing of adhesive.

5. QUESTIONS

1. What are potential safety concerns for this experiment?

2. Sketch a simple (metal wire) strain gage. On this drawing indicate where the leads should be attached. Also indicate the direction of "transverse sensitivity". Should the transverse sensitivity be high or low? What is the difference between a strain gage and a rosette?

3. Using the variables defined for the cantilevered beam in Figure 10, write down an equation for

a) Displacement δ at the loading point for an applied end load P
b) Moment as a function of position x, M(x) for an applied end load P. (Apply static equilibrium to a free-body diagram of a portion of the beam.)
c) Axial stress σ_x in terms of moment
d) Axial strain ε_x as a function axial stress (Reduce Hooke's law.)

Combine these equations to obtain axial strain $\varepsilon_x(x)$ on the upper surface of the beam as a function of x, in terms of the applied displacement δ (i.e., eliminate P from the equation.). **Use this equation during the lab, to check whether your measured strains are accurate.**

Use these equations to answer the following questions:

i) Where along the length of the beam will the <u>maximum deflection</u> occur?

ii) Where along the length of the beam will the <u>maximum linear strain</u> occur?

iii) Does this strain occur perpendicular or parallel to the axis of the beam?

4. Using the results of question 3, derive an expression for the axial strain gradient along the beam's upper surface. (Note that the strain gradient is $d\varepsilon_x/dx$.)

5. Using the results of question 3, calculate the displacement to be applied at the free end of the cantilever beam in order to produce an axial stress of approximately 35 MPa at the fixed end of the beam. Assume E = 71.7 GPa, L= 100 mm, and t = 3 mm. Compare to the value for t=7 mm. Note that, if the yield strength of the aluminum bar is S=240 MPa, then this calculated displacement is the maximum displacement before plastic deformation of the beam will occur. **Avoid plastic deformation of the beams.**