Ph.D. QUALIFICATION EXAM

The Qualifying Exam is given twice every year in December and May in four main areas (Algebra, Analysis, Applied Mathematics, Geometry-Topology). Every student must choose at least two main areas.

Exam Topics (Subtopic A, Subtopic B, Subtopic C)

1) ALGEBRA (Algebraic Number Theory, Finite Field, Grups and Rings)

2) ANALYSIS (Advaced Calculus, Functional Analysis, Real Analysis, Complex Analysis)

3) APPLIED MATHEMATICS (Numerical Analysis, Ordinary Differential Equations, Partial Differential Equations)

4) GEOMETRY AND TOPOLOGY (Differential Geometry, Algebraic Topology, Algebraic Geometry)

1) ALGEBRA

A: Algebraic Number Theory (MATH 615)

Algebraic numbers, Ring of integers of an algebraic number field, Integral bases, Norms and traces, The discriminant, Factorization into irreducibles, Euclidean domains, Dedekind domains, Prime factorization of ideals, Principal ideal rings, Lattices, Minkowski's Theorem, Geometric Representation of Algebraic Numbers, Class-group and class number, Computational Methods, Fermat's Last Theorem, Dirichlet's Units Theorem, Quadratic Residues.

- **Reference 1** Algebraic Number Theory, Serge Lang.
- **Reference 2** A Course in Computational algebraic Number Theory, Henri Cohen.

B: Finite Field (MATH 518)

Structure of finite fields, Characterization of finite dields, Roots of irreducible polynomials, Traces, norms and bases, Roots of unity and cyclotomic polynomials, Representation of elements of finite fields, Polynomials over finite fields, Order of polynomials and primitive polynomials, Irreducible polynomials, Construction of irreducible polynomials, Factorization of polynomials over finite fields, Exponential sums, Lineer recurring sequences, The minimal polynomial, Theoretical applications of finite fields, Finite geometries, Combinatorics, Linear modular systems, Pseudorandom sequences.

- **Reference 1** Introduction to finite fields and their applications, Rudolf Lidl, Harald Niederreiter.

C: Grups and Rings (MATH 501)

Groups, cyclic groups, finite groups, alternating groups, quotient groups, isomorphism theorems, direct products of groups, free groups, free abelian groups, finitely generated abelian groups, Group actions on sets, Sylow theorems. Rings, ring homomorphisms, ideals, quotient rings, factorization in commutative rings, principal ideal domains, Euclidean domains, unique factorization domains, polynomial rings, factorization in polynomial rings, power series.

- **Reference 1** Algebra, Larry C. Grove.
- **Reference 2** A First Course in Abstract Algebra, J. B. Fraleigh, Seventh Edition.
- **Reference 3** Algebra, Thomas W. Hungerford.

2) ANALYSIS

A: Advanced Calculus (MAT 211, MAT 212)

Quadric surfaces, Cylindrical and spherical coordinates, Vector functions of one variable , Curves and parametrizations (in 3-space), Functions of several variables, Limits and continuity, Partial derivatives, Higher-order derivatives, The chain rule, Linear approximations, differentiability and differentials, Gradients and directional derivatives, The gradient in three and more dimensions, Implicit functions, Extreme values, Extreme values of functions defined on restricted domains, Lagrange multipliers, Parametric problems, Double Integrals, Iteration of double integrals in Cartesian coordinates , Improper integrals and a mean-value theorem, Double integrals in polar coordinates, Change of Variables in double integrals, Triple integrals, Change of variables in triple integrals, Applications of multiple integrals, Vector and scalar fields, Conservative fields, Line integrals, line integrals of vector fields, Surfaces and surface integrals, oriented surfaces and flux integrals, Gradient, divergence and curl, Green's theorem in the plane, Divergence theorem in 3-space, Stokes's Theorems.

- **Reference 1** R. A. Adams, C. Essex, Calculus A Complete Course, Ninth edition, Pearson.
- **Reference 2** R. Silverman, Calculus with Analytic Geometry, Prentice Hall.
- **Reference 3** J. E. Marsden, M.J. Hoffman, Elementary Classical Analysis, W. H. Freeman

B: Functional Analysis (MATH 509, MATH 510)

Metric Spaces, Normed spaces, Banach spaces. Linear operators, Spaces of bounded linear operators. The uniform boundedness principle and the open mapping theorem, Bounded linear functionals. Dual spaces. The Hahn-Banach extension theorem. Separation of convex sets. Spaces of continuous functions, Ascoli's theorem, Stone-Weierstrass' theorem, Spaces of Holder continuous functions and of k-times differentiable functions, Hilbert spaces, Compact operators on a Hilbert space. Fredholm's alternative. Spectrum and eigenfunctions of a compact, self-adjoint operatör, Weak derivatives, Sobolev spaces, Embedding theorems.

- **Reference 1** Introductory Functional Analysis with Applications. Erwin Kreyszig.
- **Reference 2** Functional Analysis, Sobolev Spaces and Partial Differential Equations, Haim Brezis.
- **Reference 3** A Course in Functional Analysis, John B. Conway.

C: Real Analysis (MATH 505)

Set theory and real numbers, general measure and integration theory. Lebesgue measurable sets, measurable functions, convergence theorems, Radon-Nikodym theorem, outer measure, Carathe-odory extension theorem, product measures, Riesz representation, Baire Category, Banach Spaces. Sihned measures. Improper integrals.

- **Reference 1** Real Analysis, Halsey Royden and Patrick Fitzpatrick, Pearson; 4th edition.

- **Reference 2** Principle of Real Analysis,C.D. Aliprantis and O. Burkinshaw 3rd edition
- **Reference 3** Real analysis, G. Folland.
- **Reference 4** Real Analysis, Emmanuele DiBenedetto, Birkhauser, 2nd edition.

D: Complex Analysis (MATH 504)

Complex Numbers, Sums and Products, Basic Algebraic Properties, Vectors and Moduli. Complex Numbers, Complex conjugates, Exponential Form, Products and Powers in Exponential Form, Arguments of Products and Quotients, Roots of Complex Numbers. Regions in the Complex Plane, Functions of Complex variable, Mappings by the Exponential Function. Limits, Theorems on Limits, Limits involving the Point at infinity, Continuity. Derivatives, Differentiation Formulas, Cauchy Riemann Equations, Sufficient Conditions for Differentiability. Polar Coordinates, Analytic Functions, Harmonic Functions. Elementary Functions: The exponential function, the logarithmic function, Complex Exponents. Trigonometric Funtions, Hyperbolic functions, inverse trigonometric and hyperbolic functions, Definite integrals. Contours, Contour integrals, Upper bounds for moduli of contour integrals, Antiderivatives. Cauchy-Goursat theorem, simply connected domains, multiply connected domains, cauchy integral formula. Series: convergence of sequences, convergence of series, Taylor series, Laurent series. Series: Absolute and uniform convergence of power series, continuity of sums of power series , continuity of sums of power series, integration and differentiation of power series, uniqueness of series representation. Isolated singular points, residues, Cauchy’s residue theorem, Residue at Infinity, zeros of analytic functions. Applications of Residues.

- **Reference 1** Complex variables and Applications, J. W. Brown, R. V. Churchill.
- **Reference 2** Complex Anlaysis, S.Lang, Third Edition, 1993.
- **Reference 3** Complex Analysis, by T. Gamelin, Springer-Verlag, New York.

3) APPLIED MATHEMATICS

A: Numerical Analysis (MATH 609)

Linear Spaces, Linear Operators on Normed Spaces, Approximation Theory, Best Approximation, Finite Difference Method, Lax equivalence theorem, Sobolev Spaces, Weak derivatives, Sobolev spaces, Periodic Sobolev spaces, Weak Formulations of Elliptic Boundary Value Problems, The Lax-Milgram Lemma, The Galerkin Method and Its Variants, Finite Element Analysis, Basics of the finite element method, Error estimates of finite element interpolations Local interpolation error estimates, Global interpolation error estimates), Convergence and error estimates.

- **Reference 1** Theoretical Numerical Analysis, 3rd. Ed., Kendall Atkinson & Weimin Han, Springer Science, 2000..
- **Reference 2** Theory and Applications of Numerical Analysis, 2nd Ed., G.M.M. Phillips, Peter J. Taylor, Elsevier Science & Technology Books, 1996..
- **Reference 3** Numerical Mathematics, 2nd Ed., Alfio Quarteroni, Riccardo Sacco & Fausto Saleri, Springer Berlin Heidelberg 2007

B: Ordinary Differential Equations (MATH 515, MATH 516)

Existence and Uniqueness for the solution of Initial Value Problems. Picard Successive Approximation Method. Continuation of Solutions. Systems of Differential Equations. Properties of System Solutions. Systems with Constant and Periodic Coefficients; Higher Order Linear Differential Equations. Autonomous Systems. Systems of Nonlinear Equations. Nonlinear Systems. Bifurcation. Boundary Value Problems. Linear Differential Operators. Boundary Conditions, Existence of Solutions of Boundary Value Problems. Eigenvalues and Eigenfunctions for Linear Differential Operators. Green's Function of a Linear Differential Operator.

- **Reference 1** Ordinary differential equations, R. K. Miller, A. N. Michel, Academic Press, 1982.
- **Reference 2** Ordinary Differential Equations, J. K. Hale, Krieger publishing company, 1980
- **Reference 3** Theory of Ordinary Differential Equations, E. A. Coddington, N. Levinson,Tata McGraw-Hill Publishing Company Limited, 1987.
- **Reference 4** Differential equations and dynamical systems, L. Perko, 3rd edition, Springer, 2000.
- **Reference 5** Ordinary differential equations and dynamical systems, T. C. Sideris, Atlantis Press, 2013

C: Partial Differential Equations (MATH 523,MATH 524)

First-order equations, characteristics. Classification of second-order equations. Laplace's equation; potential theory. Green's function, maximum principles. The wave equation: characteristics, general solution. The heat equation: use of integral transforms. The course focuses on a class of concepts and techniques of functional analysis (e.g Sobolev spaces) for the solution of linear and some nonlinear PDEs. The following concepts will also be emphasized: weak derivatives, second order elliptic and parabolic equations and their weak solutions, regularity of solutions., energy estimates, maximum principles, second order hyperbolic equations.

- **Reference 1** Partial Differential Equations, Fritz John.
- **Reference 2** Elliptic Partial Differential Equations of Second Order, David Gilbarg and Neil S. Trudinger.
- **Reference 3** Lectures on Partial Differential Equations, Ivan Georgievich Petrovsky.
- **Reference 4** Partial Differential Equations: An Introduction, Walter Strauss.
- **Reference 5** Partial Differential Equations, Michael E. Taylor.

4) GEOMETRY AND TOPOLOGY

A: Differential Geometry (MATH 506)

Calculus of Euclidean maps. Parametrized Curves and Tangent Vector. Curvature, Normal and Binormal Vector. Frenet-Serret Equations and Fundamental Theorem of Space Curves. Surfaces and Parametrization. Tangent Vectors and Tangent Space. The First Fundamental Form. Surface Area. Directional Derivatives. The Weingarten Map and The 2nd Fundamental Form. Curvature and Gauss' Theorem Egregium. Geodesics. Gauss-Bonnet Theorem. Curves and surfaces. Plane curves. Geometry of hypersurfaces. Lengths and distances. Curvature. Riemannian connection. Normal coordinates, Conjugate points, Isometric immersions. Metric and geodesic completeness. Variations of the energy functional.

- **Reference 1** A First Course in Geometric Topology and Differential Geometry, Ethan D. Bloch, Modern Birkhäuser Classics, 1997.
- **Reference 2** Elementary differential geometry, B. O'Neill, second ed., Elsevier/Academic Press, Amsterdam, 2006.
- **Reference 3** Differential geometry of curves and surfaces, M. P. do Carmo, PrenticeHall, Inc., Englewood Cliffs, N.J., 1976, Translated from the Portuguese.
- **Reference 4** A First Course in Differential Geometry: Surfaces in Euclidean Space, Lyndon Woodward and John Bolton, Cambridge University Press; 1st edition .
- **Reference 5** Differential Geometry of Curves and Surfaces (Undergraduate Texts in Mathematics) 1st ed. 2016 Edition, Kristopher Tapp, Springer.

B: Algebraic Topology (MATH 624)

Homology groups of a simplicial complex, Simplices, Homology groups, Relative homology, Topological invariance of the homology groups, Simplicial approximations, The algebra of subdivision, Relative homology and the Eilenberg-Steenrod axioms, The exact homology sequence, The axioms for simplicial theory, Singular homology theory, The singular homology groups, Acyclic models, Cohomology, Relative cohomology, Cohomology rings with surfaces, Homology with coefficients, Tensor products, Homlogy with arbitrary coefficients, Homological algebra, Torsion products, Tensor product of chain complexes.

- **Reference 1** Elements of Algebraic Topology, J. R. Munkres, Addison-Wesley Publishing Company, Inc.
- **Reference 2** Algebraic Topology, C. R. F. Maunder, Cambridge University Press.
- **Reference 3** Topology and Geometry, G. E Bredon, Springer-Verlag

C: Algebraic Geometry (MATH 622)

Guiding problems, Division algorithm and Gröbner bases, Affine varieties, Elimination, Resultants, Irreducible varieties, Nullstellensatz, Primary decomposition, Projective geometry, Projective elimination theory, Parametrizing linear subspaces, Hilbert polynomials and the Bezout Theorem.

- **Reference 1** Introduction to Algebraic Geometry, Brendan Hassett, Cambridge University Press
- **Reference 2** Using Algebraic Geometry, D. Cox, J. Little, D. O'Shea, Springer.
- **Reference 3** Algebraic Geometry, J. Harris, springer-Verlag.